Klop counter example in the Rho-Calculus

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From Klop's counter example

1 Types

The unique type constant we will use is ι. The types of the constants and variables we use are the following:

$\vdash\ rec : (\iota \rightarrowtriangle \iota \rightarrowtriangle \iota) \rightarrowtriangle \iota$
$\vdash\ rec' : (\iota \rightarrowtriangle \iota) \rightarrowtriangle \iota$
$\vdash\ d : \iota \rightarrowtriangle \iota \rightarrowtriangle \iota$
$\vdash\ e : \iota$
$\vdash\ S : \iota \rightarrowtriangle \iota \rightarrowtriangle \iota$
$\vdash\ S' : \iota \rightarrowtriangle \iota$
$\vdash\ X : \iota$
$\vdash\ Y : \iota$

2 Recursors

We define $\mathcal{C} \equiv rec(S) \rightarrowtriangle X \rightarrowtriangle (d(Y,Y) \rightarrowtriangle e)\ d(X, S.rec\ X)$
$\mathcal{A} \equiv rec'(S') \rightarrowtriangle \mathcal{C}.rec\ S'.rec'$

Thus we have:

$\vdash\ \mathcal{C} : \iota \rightarrowtriangle \iota \rightarrowtriangle \iota$
$\vdash\ \mathcal{C}.rec : \iota \rightarrowtriangle \iota$
$\vdash\ \mathcal{A} : \iota \rightarrowtriangle \iota$
$\vdash\ \mathcal{A}.rec' : \iota$
$\vdash\ \mathcal{C}.rec\ \mathcal{A}.rec' : \iota$

And the following reductions:

$$\begin{array}{ccccc}
\mathcal{A}.rec' & \longrightarrow & \mathcal{C}.rec\ \mathcal{A}.rec' & \longrightarrow & (d(Y,Y) \rightarrowtriangle e)\ d(\mathcal{A}.rec', \mathcal{C}.rec\ \mathcal{A}.rec') \\
 & & \downarrow\!\!\downarrow & & \downarrow \\
 & & \mathcal{C}.rec\ e & & (d(Y,Y) \rightarrowtriangle e)\ d(\mathcal{C}.rec\ \mathcal{A}.rec', \mathcal{C}.rec\ \mathcal{A}.rec') \\
 & & & & \downarrow \\
 & & & & e
\end{array}$$

e is in normal form (it is a constant) and the only possible reduction from $\mathcal{C}.rec\ e$ is $\mathcal{C}.rec\ e \mapsto (d(Y,Y) \rightarrowtriangle e)\ d(e, \mathcal{C}.rec\ e)$ and the first abstraction can be dropped only if e and $\mathcal{C}.rec\ e$ have a common reduction. We conclude by induction on the supposed length of a reduction from $\mathcal{C}.rec\ e$ to e.