Matching modulo superdevelopments
Application to second-order matching

Germain Faure

Ws Rho'06

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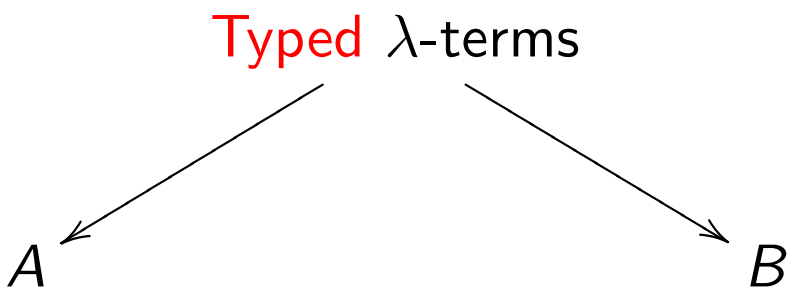

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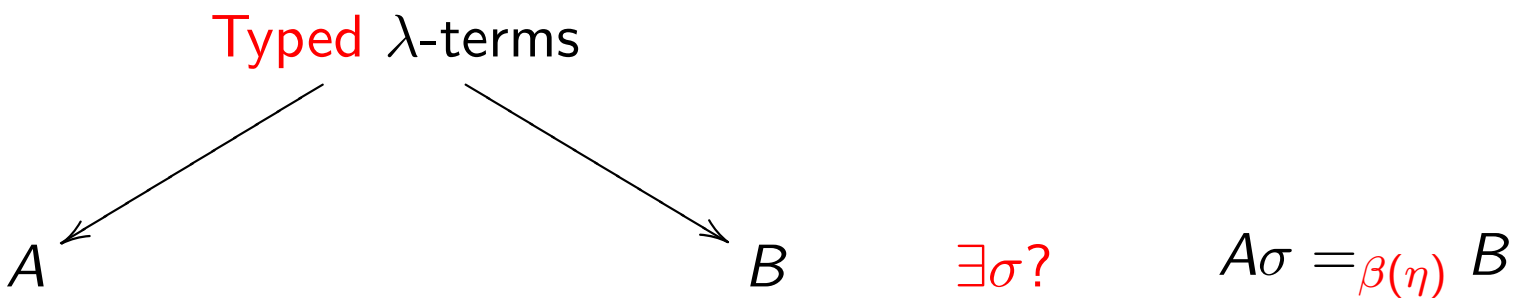

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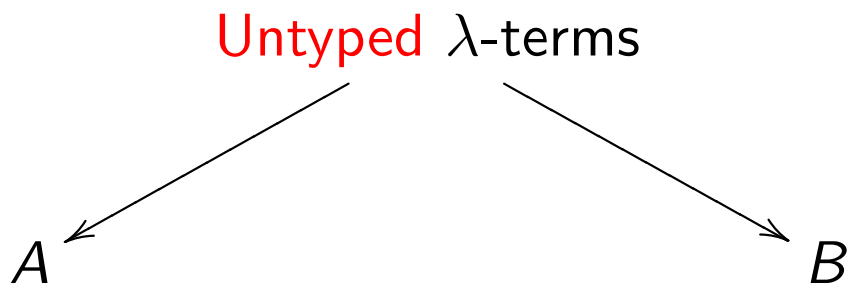

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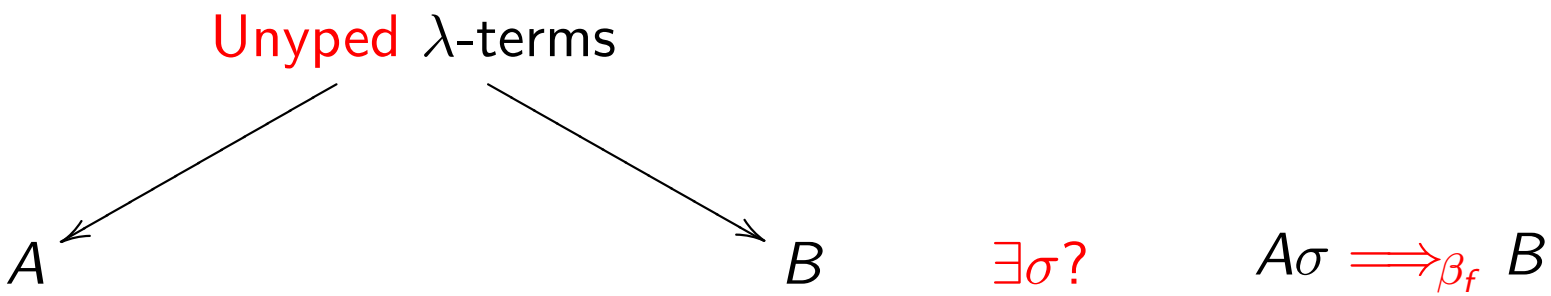

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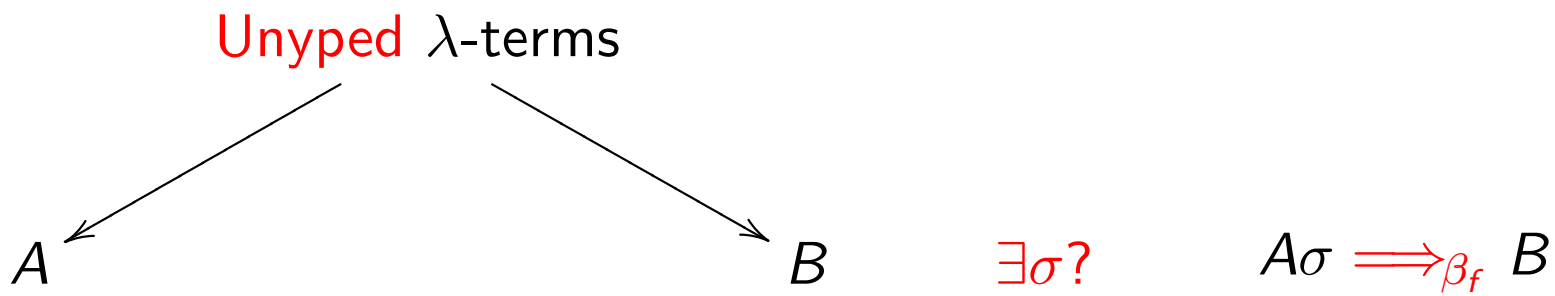

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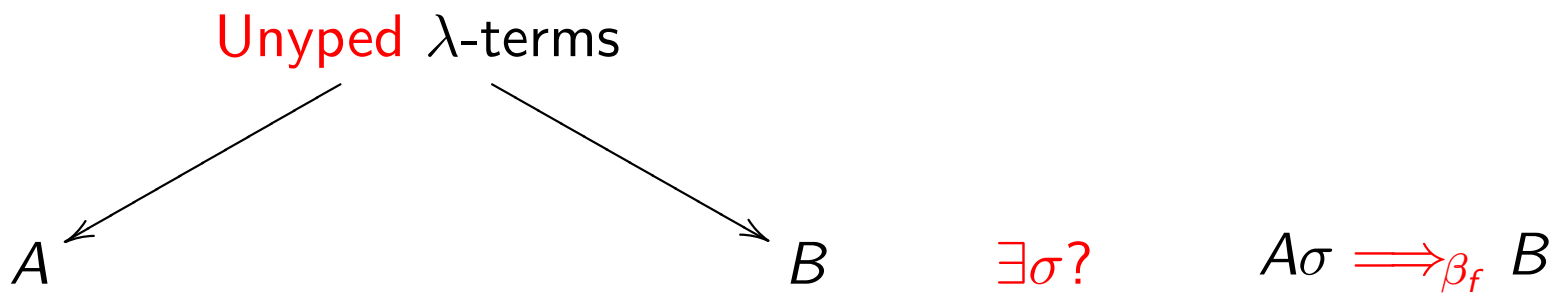

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  - that gives an approximation of β-normal form

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  ➥ Example: Pattern-matching in the ρ-calculus

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   - Transformations of programs with built-in pattern-matching (not only purely functional programs)
   - Proof theory that handles rich proof-terms in the generalized deduction modulo [**?**]

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➥ Intuition
Deeper understanding

Road-map

1. Superdevelopments. Strong parallel β-reduction.
2. Matching modulo SD. Links with other h.o. matching algorithms.
3. Algorithm for matching modulo SD.
4. Second-order matching.
5. The η-equivalence.

The λ-calculus

- Syntax

A, B, C	$::=$	x	variable
	$\mid$	X	matching variable
	$\mid$	c	constant
	$\mid$	$\lambda x.A$	abstraction
	$\mid$	AB	application

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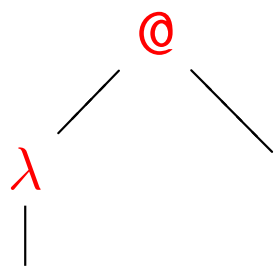
- Operational semantics

$$(\lambda x.A)B \qquad \rightarrow_\beta \qquad A[x := B]$$

Creation of redexes in the λ-calculus (I)



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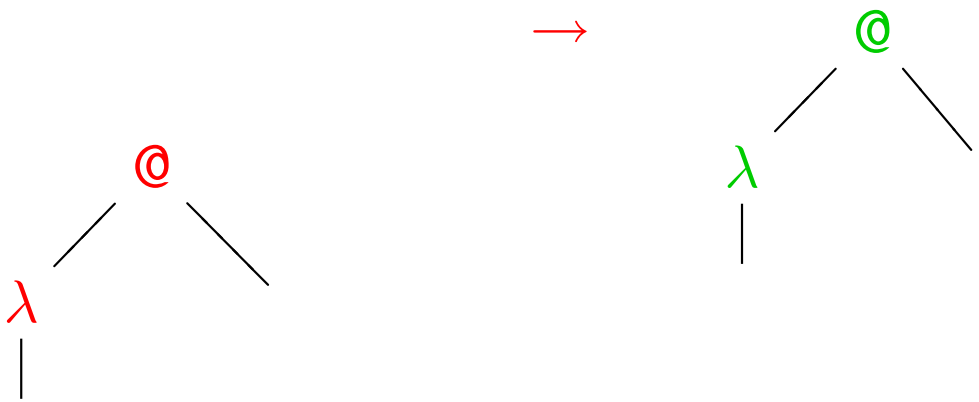

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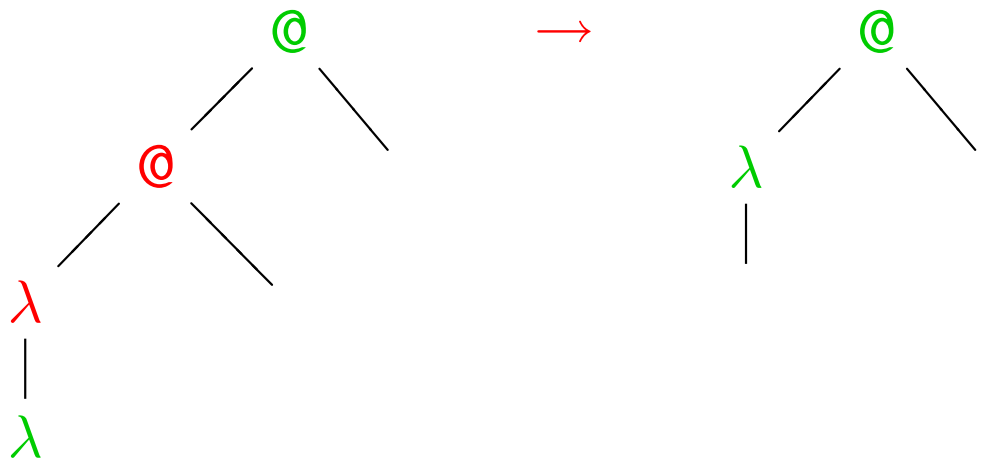

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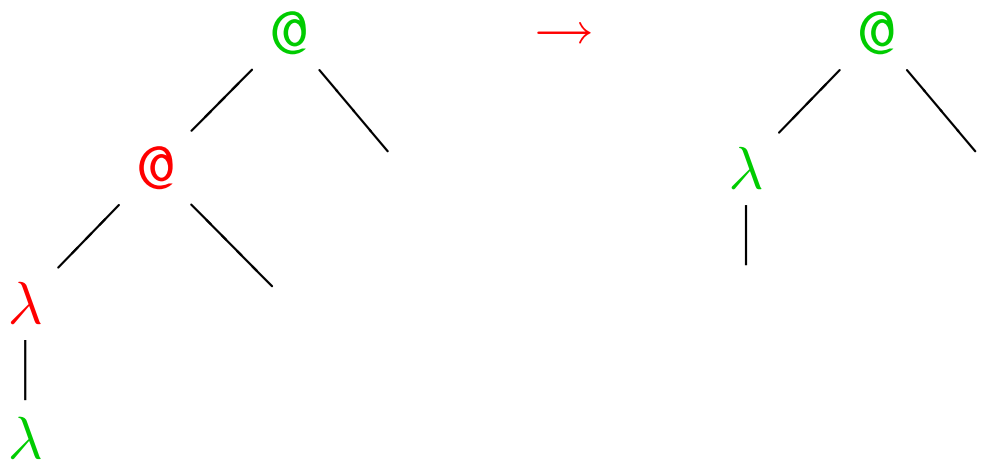
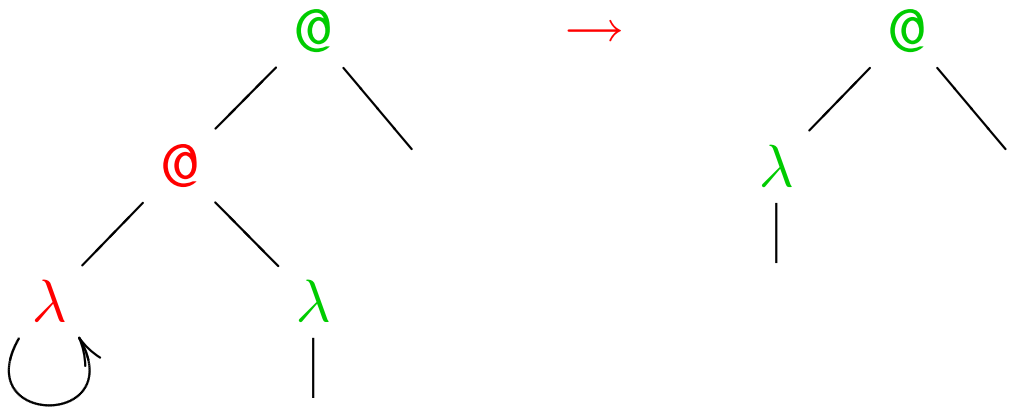


$((\lambda x.\lambda y.x)\ a)\ b \quad \rightarrow \quad (\lambda y.a)\ b$

Creation of redexes in the λ-calculus (II)



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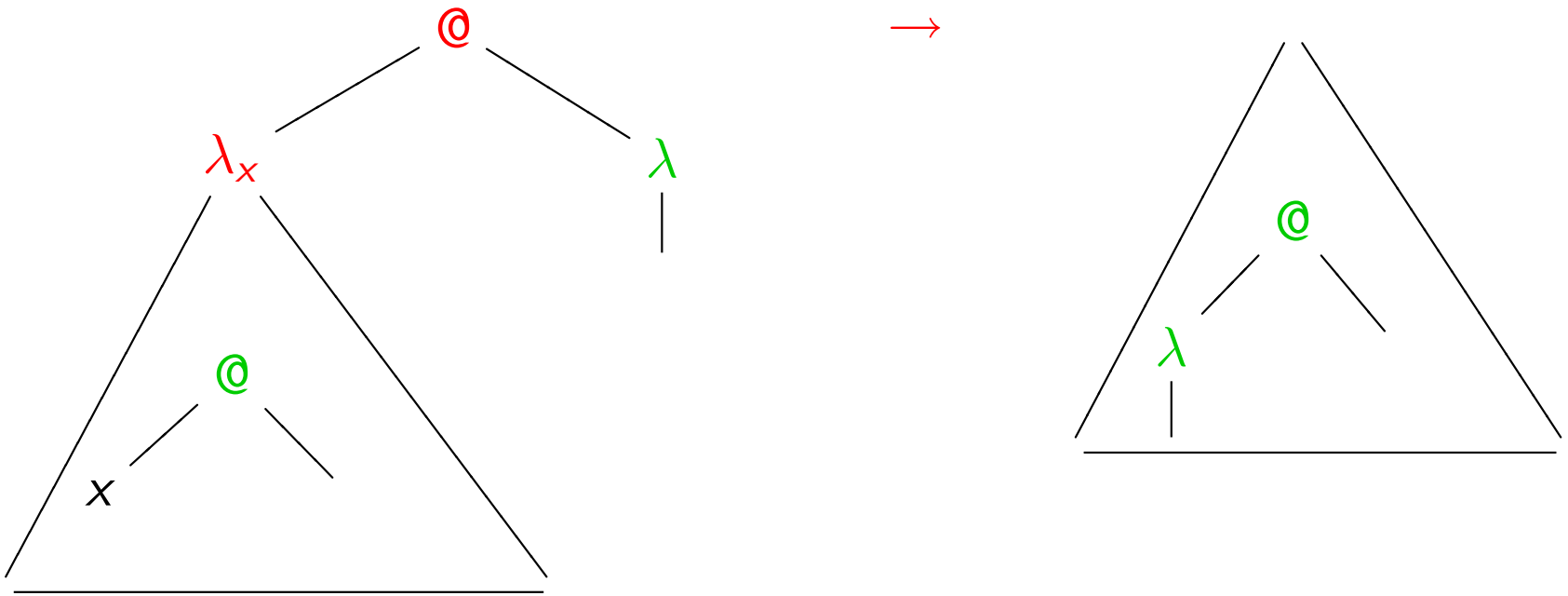

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Definition [?]

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Examples of superdevelopments

Couter-examples of superdevelopments

Other characterization

Superdevelopments

The residuals of the redexes can be reduced as in developments

Example

$$
\begin{array}{ll}
(\lambda x.f(x,x))\,((\lambda y.y)\,a) & \\
\rightarrow_\beta & f((\lambda y.y)\,a, (\lambda y.y)\,a) \\
\rightarrow_\beta & f(a, (\lambda y.y)\,a) \\
\rightarrow_\beta & f(a,a)
\end{array}
$$

Superdevelopements

The following β-reduction is a superdevelopement:

Example

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The parallel β-reduction [Tait Martin-Löf]

$$(\textsc{Red}\text{-}\beta)\ \frac{\lambda x.A_1 \Longrightarrow_\beta \lambda x.A_2 \qquad B_1 \Longrightarrow_\beta B_2}{(\lambda x.A_1)B_1 \Longrightarrow_\beta A_2[x := B_2]}$$

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The strong parallel β-reduction [?]

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$$(\textsc{Red}\text{-}\aleph)\ \frac{}{\aleph \Longrightarrow_{\beta_f} \aleph}$$

Equivalence between superdevelopments

Theorem [?]

There exists a superdevelopment $A \mapsto\!\!\!\!\twoheadrightarrow_{\beta} B$

iff

$A \Longrightarrow_{\beta_f} B$

Matching in the pure λ-calculus modulo SD

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Definition [β_{sd}-matching system]

a multiset of matching equations

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A substitution is a match of a system if it matches each equation

Solved form

Definition

- For a matching equation $X \leqslant_{\beta_{sd}} A$: no variables in A
- For a system: each matching variable occurs once

Matching in the simply typed λ-calculus modulo β

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Definition [β-equation]

a pair of β-normal typed λ-terms denoted $A \ll_\beta B$

- of the same type

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We can associate to a β-match equation, a β_{sd}-match equation (erase types)

An example

The equation $X^{(\iota\to\iota)\to\iota}\, Y^{(\iota\to\iota)} \ll_\beta a^{\kappa\to\iota\to\iota}(b^\kappa, c^\iota)$

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$X \leftarrow \lambda x.a(b,c)$	✓	✓

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$X \leftarrow \lambda x.a(b,c)$	✓	✓
$X \leftarrow ab \qquad Y \leftarrow c$	✓	✗

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$X \leftarrow \lambda x.a(b,c)$		✓	✓
$X \leftarrow ab$	$Y \leftarrow c$	✓	✗
$X \leftarrow \lambda x.a(b,xc)$	$Y \leftarrow \lambda y.y$	✗	✓

SD and second-order matching

Proposition

Let ϕ be a β-match a second-order β-matching equation. Then it is also a β_{sd}-match.

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Proof In (III) there is a 3^{rd}-order redex

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  - ➥ Np-completeness of second-order matching ?

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Proposition

Let P be a pattern. If ϕ is a β-match of $P \ll_\beta B$ then

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- **Proof:** Patterns do not need full β-conversion but only β_0-conversion:

$$(\lambda x.t)x \quad \rightarrow \quad t$$

and this reduction does not create redexes in (III).

Algorithm for matching modulo superdevelopments

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$$\overline{A\sigma \Longrightarrow_{\beta_f} B}$$

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$$(X \leqslant_{\beta_{sd}} B),\ \mathbb{S} \quad \rightarrow_{\aleph_X} \quad X \leqslant_{\beta_{sd}} B,\ \mathbb{S}\{A/X\}$$

if $\mathcal{FV}(B) = \emptyset$ **and** $X \in \mathbb{S}$

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(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \rightarrow_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots
\end{array}
$$

$$
\frac{}{(\lambda x.A_1)\sigma \Longrightarrow_{\beta_f}}
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$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \rightarrow_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \rightarrow_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \rightarrow_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots
\end{array}
$$

$$
\frac{}{(\lambda x.A_1)\sigma \Longrightarrow_{\beta_f} \lambda x.B_1}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A \leqslant_{\beta_{sd}} \lambda x.B),\ \mathbb{S} & \to_{\lambda_\lambda} & (A \leqslant_{\beta_{sd}} B),\ \mathbb{S}
\end{array}
$$

$$
\frac{}{(\lambda x.A_1)\sigma \Longrightarrow_{\beta_f} \lambda x.B_1}
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Algorithm for matching modulo superdevelopments

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\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S}
\end{array}
$$

$$
\frac{}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f}}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1 B_2}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1 B_2}
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Algorithm for matching modulo superdevelopments

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\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f}}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1[x := B_2]}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1[x := B_2]}
$$

$x \in B_1$?

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \rightarrow_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \rightarrow_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \rightarrow_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \rightarrow_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \rightarrow_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1}
$$

$x \notin B_1$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_x} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} & \to_{@_\pi} & A_1 \leqslant_{\beta_{sd}} \lambda x.B_1 \\
 & & \text{if } x \notin B_1
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1}
$$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \rightarrow_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \rightarrow_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \rightarrow_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \rightarrow_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \rightarrow_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} & \rightarrow_{@_\pi} & A_1 \leqslant_{\beta_{sd}} \lambda x.B_1 \quad \textbf{if} \ldots
\end{array}
$$

$$(A_1 A_2)\sigma \Longrightarrow_{\beta_f}$$

$x \in B_1$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} & \to_{@_\pi} & A_1 \leqslant_{\beta_{sd}} \lambda x.B_1 \quad \textbf{if} \ldots
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1[x := B_2]}
$$

$x \in B_1$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} & \to_{@_\pi} & A_1 \leqslant_{\beta_{sd}} \lambda x.B_1 \quad \textbf{if} \ldots \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1[x := B_2]),\ \mathbb{S} & \to_{@_\beta} & (A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S}
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1[x := B_2]}
$$

$x \in B_1$

Algorithm for matching modulo superdevelopments

$$
\begin{array}{lll}
(a \leqslant_{\beta_{sd}} a),\ \mathbb{S} & \to_{\aleph_c} & \mathbb{S} \\
(x \leqslant_{\beta_{sd}} x),\ \mathbb{S} & \to_{\aleph_v} & \mathbb{S} \\
(X \leqslant_{\beta_{sd}} A),\ \mathbb{S} & \to_{\aleph_X} & X \leqslant_{\beta_{sd}} A,\ \mathbb{S}\{A/X\} \quad \textbf{if} \ldots \\
(\lambda x.A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ \mathbb{S} & \to_{\lambda_\lambda} & (A_1 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1 B_2),\ \mathbb{S} & \to_{@_@} & (A_1 \leqslant_{\beta_{sd}} B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
(A_1 A_2 \leqslant_{\beta_{sd}} B_1),\ \mathbb{S} & \to_{@_\pi} & A_1 \leqslant_{\beta_{sd}} \lambda x.B_1 \quad \textbf{if} \ldots \\
(A_1 A_2 \leqslant_{\beta_{sd}} C),\ \mathbb{S} & \to_{@_\beta} & (A_1 \leqslant_{\beta_{sd}} \lambda x.B_1),\ (A_2 \leqslant_{\beta_{sd}} B_2),\ \mathbb{S} \\
 & & \textbf{where } C = B_1[x := B_2] \\
 & & \textbf{and } x \in B_1
\end{array}
$$

$$
\frac{A_1\sigma \Longrightarrow_{\beta_f} \lambda x.B_1 \qquad A_2\sigma \Longrightarrow_{\beta_f} B_2}{(A_1 A_2)\sigma \Longrightarrow_{\beta_f} B_1[x := B_2]}
$$

Example: $XY \leqslant_{\beta_{sd}} ab$

- Rule @@: $X \leqslant_{\beta_{sd}} a,\ Y \leqslant_{\beta_{sd}} b$

Example: $XY \leqslant_{\beta_{sd}} ab$

- Rule $@_{@}$: $X \leqslant_{\beta_{sd}} a,\ Y \leqslant_{\beta_{sd}} b$
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Example: $XY \leqslant_{\beta_{sd}} ab$

- Rule $@_{@}$: $X \leqslant_{\beta_{sd}} a,\ Y \leqslant_{\beta_{sd}} b$
- Rule $@_{\pi}$: $X \leqslant_{\beta_{sd}} \lambda x.ab$
- Rule $@_{\beta}$:
  -
  -
  -

Example: $XY \leqslant_{\beta_{sd}} ab$

- Rule $@_@$: $X \leqslant_{\beta_{sd}} a,\ Y \leqslant_{\beta_{sd}} b$
- Rule $@_\pi$: $X \leqslant_{\beta_{sd}} \lambda x.ab$
- Rule $@_\beta$:
  - $Y \leqslant_{\beta_{sd}} ab$
  - $Y \leqslant_{\beta_{sd}} a$
  - $Y \leqslant_{\beta_{sd}} b$

Example: $XY \leqslant_{\beta_{sd}} ab$

- Rule $@_@$: $X \leqslant_{\beta_{sd}} a,\ Y \leqslant_{\beta_{sd}} b$
- Rule $@_\pi$: $X \leqslant_{\beta_{sd}} \lambda x.ab$
- Rule $@_\beta$:
  - $X \leqslant_{\beta_{sd}} \lambda x.x,\quad Y \leqslant_{\beta_{sd}} ab$
  - $X \leqslant_{\beta_{sd}} \lambda x.xb,\ Y \leqslant_{\beta_{sd}} a$
  - $X \leqslant_{\beta_{sd}} \lambda x.ax,\ Y \leqslant_{\beta_{sd}} b$

Example: $X(YX) \leqslant_{\beta_{sd}} a$

- Rule $@_\pi$: $X \leqslant_{\beta_{sd}} \lambda x.a$

Example: $X(YX) \leqslant_{\beta_{sd}} a$

- Rule $@_\pi$: $X \leqslant_{\beta_{sd}} \lambda x.a$
- Rule $@_\beta$: $X \leqslant_{\beta_{sd}} \lambda x.x,\ YX \leqslant_{\beta_{sd}} a$

Example: $X(YX) \leqslant_{\beta_{sd}} a$

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Example: $X(YX) \leqslant_{\beta_{sd}} a$

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Example: $X(YX) \leqslant_{\beta_{sd}} a$

- Rule $@_\pi$: $X \leqslant_{\beta_{sd}} \lambda x.a$
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Example: $X(YX) \leqslant_{\beta_{sd}} a$

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    - Rule $\aleph_X$: $X \leqslant_{\beta_{sd}} \lambda x.x,\ Y \leqslant_{\beta_{sd}} \lambda x.x,\ \lambda x.x \leqslant_{\beta_{sd}} a$

      ➥ Not a solved form

Main properties

- No new matching variables

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- Termination

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- Completeness

Main properties

- No new matching variables
- Termination
- Soundness
- Completeness
- Finite complete match set

Back to second-order

Applying the previous algorithm in a typed context:

We get a new algorithm for second-order matching

Matching modulo sd and η

The customization is easy

Matching modulo sd and η

The customization is easy

- Only generate $\beta\eta$-normal forms for the rule $@_\beta$

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Matching modulo sd and η

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$$\lambda x.A \leqslant^{\eta}_{\beta_{sd}} B,\ \mathbb{S} \qquad \rightarrow_{\lambda\_} \qquad A \leqslant^{\eta}_{\beta_{sd}} Bx,\ \mathbb{S}$$

if B is not an abstraction
and x fresh

Matching modulo sd and η

The customization is easy

- Only generate $\beta\eta$-normal forms for the rule $@_\beta$
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$$\lambda x.A \leqslant^{\eta}_{\beta_{sd}} B,\ \mathbb{S} \qquad \rightarrow_{\lambda\_} \qquad A \leqslant^{\eta}_{\beta_{sd}} Bx,\ \mathbb{S}$$

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and x fresh

Proposition

In the context of patterns *à la* Miller, this algorithm gives the most general match.

Conclusion

- A new approach to higher-order matching:

Conclusion

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Conclusion

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  - Complete for second-order
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  - Use of η-equivalence is optional
- The simple algorithms
  - properties: termination, soudness, completeness

Conclusion

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Conclusion

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- Higher-order matching in the ρ-calculus